Calibration of Modified Cam-Clay Parameters for Red Mud Tailings – A Case Study

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Abstract

Critical state soil mechanics proposes an integrated approach to soil behaviour, introducing the concept of critical state associated with the theory of hardening and softening plasticization. The critical state approach shows a more rigorous treatment in solving problems, especially those related to stress-strain behaviour. The dissemination of this concept is still relatively new in Brazil, especially among geotechnical engineers in the mining industry, where the limit equilibrium has been the main method to evaluate the factor of safety of the geotechnical structures for decades. This makes it necessary to develop studies that promote discussions about the benefits of the critical state approach. Therefore, this study presents the calibration of the Modified Cam-Clay parameters for red mud tailings and its evaluation by comparing the behaviour simulated by the model with i) the results of drained and undrained isotropically consolidated triaxial compression tests; and ii) the behaviour simulated by the widely used Mohr-Coulomb model. This study also shows an interpretation of the variation of the red mud behaviour with the variation of the degree of compaction of a trial dry stack assisted by the calibration of the Modified Cam-Clay model. The input parameters were defined after oedometer and triaxial compression tests, and GeoStudio© (SIGMA/W) was used for the numerical modeling. When compared to the triaxial compression tests, the Modified Cam-Clay model simulated the behaviour of the red mud tailings in drained and undrained conditions. The loose samples converged to an OCR lower than 2.5, and the dense samples to an OCR between 5 and 8, which showed that the dilatancy was simulated with the OCR variation. The calibration confirmed the limitations of the Mohr-Coulomb model in terms of stress-strain response. Unlike the Modified Cam-Clay, the Mohr-Coulomb model could not simulate adequately the strain rates and porewater pressure generation of the studied conditions. This highlights the importance of adopting the proper constitutive model in stress-strain studies. Finally, for the studied stress conditions, a strong dilative behaviour simulated by high OCR in the model was only identified for samples with an initial void ratio (prior consolidation) lower than 0.9 (degree of compaction of 104.5% of Standard Proctor). This suggests that the degree of compaction in the field

could be adjusted to this value to minimize the deformations and the risk of failure of the structure.

Introduction

The critical state soil mechanics (CSSM) proposes an integrated approach for soil behaviour, correlating deformability and resistance aspects, which are treated separately in classical soil mechanics.

The correlation between deformability and resistance is associated with a tridimensional yield surface involving q, p', and e, which can be represented through mathematical equations described in elastoplastic models, in which the deformations can be treated in elastic and plastic domains separately.

Casagrande (1936), when evaluating the behaviour of sands in loose and dense states through direct shear tests in drained conditions, observed that loose sands, when sheared, presented contractive behaviour, and those in dense conditions indicated dilatant behaviour. At large strains (above 10%), Casagrande (1975) observed that the materials presented the same void ratio, which the author classified as the critical void ratio (e_c), considering the performance of drained tests with 100 kPa normal stress for three states conditions: i) loose sand; ii) dense sand and iii) sample with critical void ratio. Roscoe et al. (1958) defined the critical state as the state at which particulate materials, such as the soils, continue to deform at constant stress and constant void ratio – essentially a formalization of Casagrande's idea.

Taylor (1948) observed that the critical void ratio is affected by the mean effective stress (p'), becoming smaller with increasing stress applied to the soil mass. The author evaluated the soil behaviour during the plastic phase and observed that the relationship between the final void ratio and the logarithm of the applied stress could be given as a straight line parallel and slightly inferior to the normal compression line (NCL), classified as the ultimate condition of the material and named as critical state line (CSL).

Castro (1969) undertook a series of stress-controlled triaxial tests to reproduce field loading conditions which Casagrande surmised were stress controlled. These tests on loose samples resulted in liquefaction failures leading to a well-defined steady state at the end of the tests. Been et al. (1991) examined in detail the difference between the critical and steady-state lines and concluded that, for practical purposes, equivalence could be assumed.

The representation of the CSL on the $q - p'$ and $e - p'$ planes can be represented by Equation 1 and Equation 2, respectively.

$$q = Mp' \quad (1)$$

$$e_c = \Gamma - \lambda ln(p'_c) \quad (2)$$

Where q is the deviator stress, M is the critical friction ratio, Γ is the "altitude" of CSL defined at 1 kPa, λ is the slope of CSL and p'_c is the critical mean effective stress.

Additionally, through triaxial compression tests, M can be defined according to Equation 3.

$$M = \frac{6 \sin \varphi'_c}{3 - \sin \varphi'_c} \quad (3)$$

Where φ'_c is the effective friction angle at the critical state.

After the recent tailings dam failures in Brazil, namely Fundão Dam (2015) and Brumadinho B-I Dam (2019), many mining companies have been implementing the dry stack to dispose of tailings. More straightforward solutions are often adopted for stress-strain studies of this type of tailings storage facilities, which raises the necessity to develop studies capable of promoting discussions about the benefits of the approaches from the critical state theory.

In this context, this paper aims to present the calibration of the Modified Cam-Clay (MCC) parameters for red mud tailings and its evaluation by comparing the behaviour simulated by the model with i) the results of drained and undrained isotropically consolidated triaxial compression tests; and ii) the behaviour simulated by the widely used Mohr-Coulomb (MC) model. Additionally, this study shows an interpretation of the variation of the red mud behaviour with the variation of the degree of compaction of a trial dry stack assisted by the calibration of the Modified Cam-Clay model.

Red mud characterization

The tailing evaluated in this study is the by-product of alumina refining. As indicated in Figure 1, the average tailings composition (ASTM D422-63) is 9.4% sand, 32.3% silt and 58.3% clay size, with an average specific gravity (*Gs*) of 3.14. On average, the Atterberg limits, determined according to ABNT NBR 6459 and ABNT NBR 7180, indicated a liquid limit (LL) of 34% and a plastic limit (LP) of 25% (plasticity index of 9%), as well as a maximum dry unit weight ($\gamma_{d,max}$) of 15.5 kN/cm^3 and an optimum moisture content (w_{opt}) of 28.2%, considering the Standard Proctor.

For the study, undisturbed block samples were collected in different areas of a trial dry stack. Three oedometer tests were performed, and the results indicated an NCL slope (λ) of 0.128 and a slope of the recompression/swelling line (k) between 0.007 and 0.0174, as shown in Figure 2. Nine drained (six loose and three dense specimens) and nine undrained (five loose and four dense specimens) isotropically consolidated triaxial compression tests were performed from the undisturbed block samples collected.

The specimens were consolidated to 50 kPa, 150 kPa, 300 kPa, and 600 kPa. The critical state line (CSL) was defined after the drained and undrained triaxial tests on the loose specimens. Figure 3 presents the CSL plotted on the $e - p'$ and $q - p'$ planes and the tests results used to define it. A scatter of the results was observed, which may be associated with the variability of the red mud since the samples were collected in different areas of the dry stack, and the quality of the data due to the limitations of the triaxial test apparatus, not in conformity with all the specifications proposed by Jefferies and Been (2016), Reid et al. (2021), and Viana da Fonseca et al. (2021). Despite the test limitations, the CSL defined on the $e - p'$ plane presented a very similar slope to the NCL defined on the $e - \sigma'_v$ plane, indicating the parallelism between both lines.

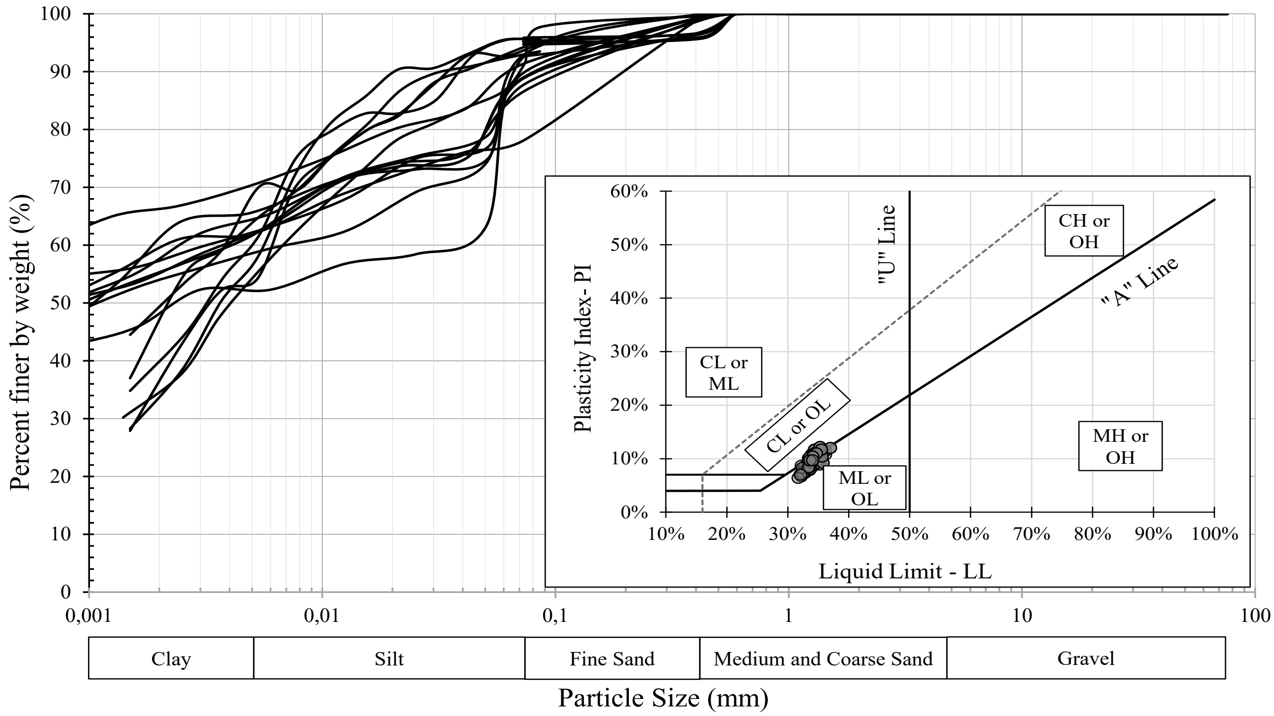

Figure 1: Grain size distribution curves (ASTM D422) and the Casagrande Plasticity Chart

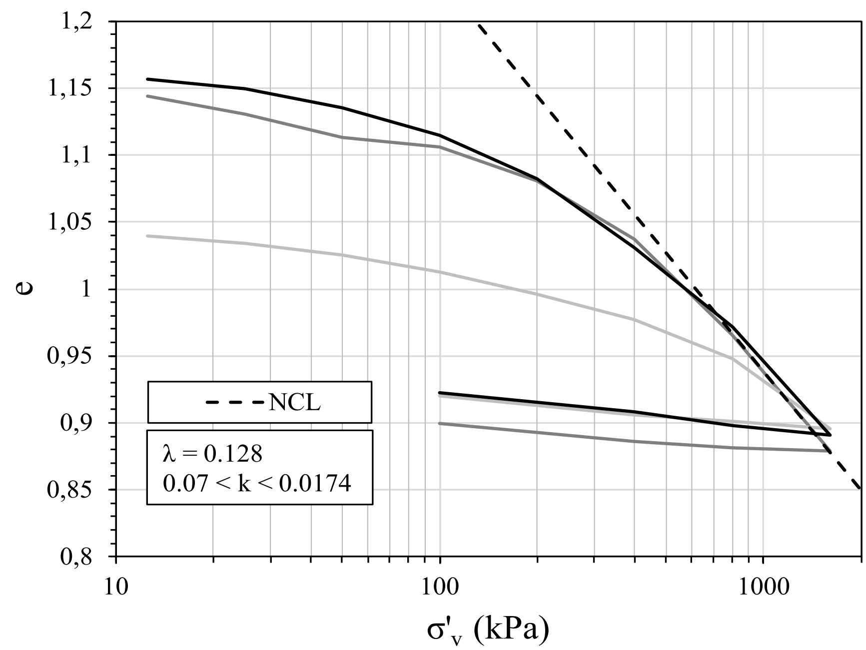

Figure 2: Normal consolidation line and recompression lines obtained from oedometer tests

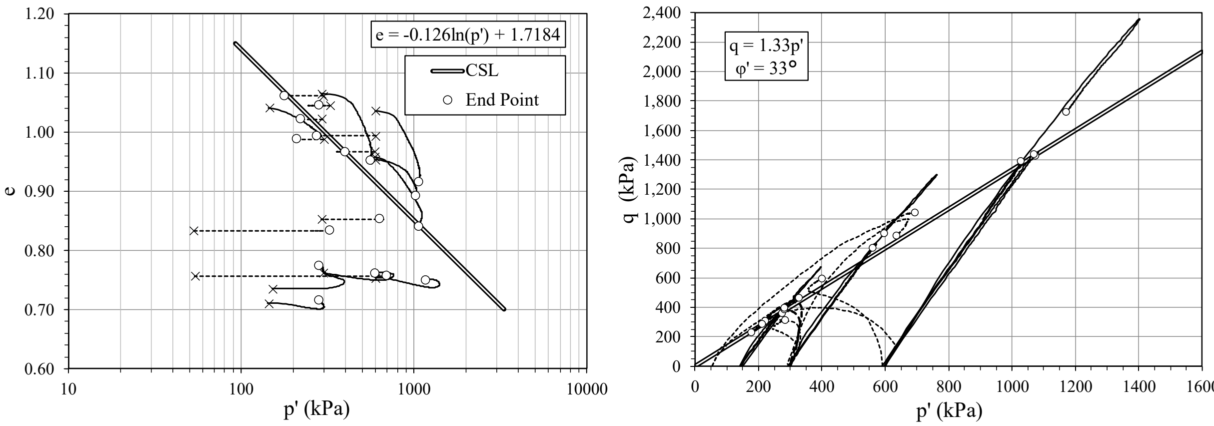

Figure 3: CSL on the *e – p'* and *q – p'* planes

Constitutive models parametrization

Mohr-Coulomb

The Mohr-Coulomb is the most common constitutive model to reproduce the behaviour of geomaterials and soils in numerical analyses. The model assumes that the material will only yield when reaching the failure plane, defined as a linear relationship between the shear (τ) and effective normal (σ') stresses, as presented by Equation 4.

$$\tau = c' + \sigma' \tan(\varphi') \qquad (4)$$

Where c' is the effective cohesion intercept and φ' is the effective friction angle.

The yield/failure surface, in terms of principal stress, is given according to Equation 5.

$$F_s(\sigma) = (\sigma_1 - \sigma_3) - (\sigma_1 + \sigma_3) \cdot \sin(\varphi') - 2c' \cdot \cos\varphi' \qquad (5)$$

Where σ_1 is the major principal stress and σ_3 is the minor principal stress.

This function represents a cross-section of an irregular hexagonal pyramid in the principal stresses space, which is fixed in the stresses space and does not change in size with additional plastic deformations. Thus, the model is typically described as linear elastic, perfectly plastic, in which the elasticity is defined by Hooke's law.

One of the limitations of this model is that the Mohr-Coulomb parameters are only related to the strength of the soil. However, it is also important to consider the behaviour in terms of deformability/compressibility. Another limitation is that the perfectly plastic response becomes unfeasible since the volumetric deformation related to the plastic portion continues to develop indefinitely with additional shear.

Table 1 presents the parameters defined after characterization to model the red mud with Mohr-Coulomb. For each analysis, the bulk unit weight (γ) and the initial void ratio (e_0) were defined considering the initial state of the samples, and the effective Young's modulus was adjusted to obtain the best calibration. The critical state friction angle ($\varphi_c{}' = 33°$) was adopted to define the failure surface.

Table 1: Mohr-Coulomb model parameters

Parameters	Values
e_0	0.77 to 1.23
γ (kN/m³)	20.10 to 19.63
E' (MPa)	30 to 60
v	0.25
c' (kPa)	0
ϕ' (°)	33

Modified Cam-Clay

Cam-Clay was the first model to consider the CSSM, along with the concepts of the plasticity theory by Drucker et al. (1957).

According to Schofield and Wroth (1968), the model could be applied to isotropic continuous materials with elastoplastic behaviour, which cannot exhibit discontinuities or overlaps during deformations. It could explain the contractive behaviour of normally to slightly over-consolidated (OCR ≤ 2.72) clays without considering the occurrence of cementation due to the clay mineral particles (Roscoe and Burland, 1968).

After the definition of the original Cam-Clay, the possibility of plastic distortional strain increments for mean effective stress increments was observed in the condition of stress ratio $\eta = 0$ (where $\eta = q/p'$). Additionally, the equations initially proposed for the model overestimated the values of strain increment for the initial stages of the triaxial compression and the values of the coefficient of earth pressure at rest (K_0). A new elliptical yield surface (Equation 6) was proposed by Roscoe and Burland (1968) to correct the abovementioned points, and the so-called Modified Cam-Clay model was defined.

$$\frac{p'}{p'_y} = \frac{M^2}{M^2 + \eta^2} \quad (6)$$

Where p'_y is the mean effective stress on the yield surface.

Failure and yield surfaces are not coincident in the Modified Cam-Clay, and the yield surface is allowed to expand due to plastic strains. The failure surface, defined by the critical state and implemented by adopting the Druker-Prager criterion, overestimate the strength in triaxial extension (Bishop 1966), representing a limitation of the model in such stress conditions.

Table 2 presents the parameters defined after characterization to model the red mud with the Modified Cam-Clay. For each analysis, the γ and e_0 were defined considering the initial state of the samples, and the OCR was adjusted for each analysis to obtain the best calibration.

Table 2: Modified Cam-Clay model parameters

Parameters	Values
e_0	0.77 to 1.23
γ (kN/m^3)	20.10 to 19.63
OCR	1 to 8
λ	0.128
k	0.0174
ν	0.25
ϕ'_c (°)	33
M	1.33

Numerical analysis methodology

After the characterization and parametrization of the Mohr-Coulomb and Modified Cam-Clay models, the numerical analyses of the material behaviour under axisymmetric compression were performed by the Finite Element Method (FEM) in the GeoStudio© SIGMA/W. The triaxial compression, which consists of a three-dimensional axisymmetric solid subject to an axisymmetric load, was reduced to a simple two-dimensional problem. The adopted geometry was defined as ¼ of the initial geometry of the specimens.

Each analysis was subdivided into two phases: consolidation and shear. In the consolidation phase, because Mohr-Coulomb is an isotropic elastic material, it was adopted along with loading boundary conditions equivalent to the consolidation stresses of each test (50 kPa, 150 kPa, 300 kPa and 600 kPa) to simulate the stress state of consolidation and define the initial stress state of the shear phase. After the consolidation phase, the accumulated displacements and strains were reset, and the respective constitutive models (MC or MCC) were applied to the models. A strain-controlled boundary condition was defined to simulate the axial loading in the shear phase. Figure 4 shows the methodology used in the numerical modeling.

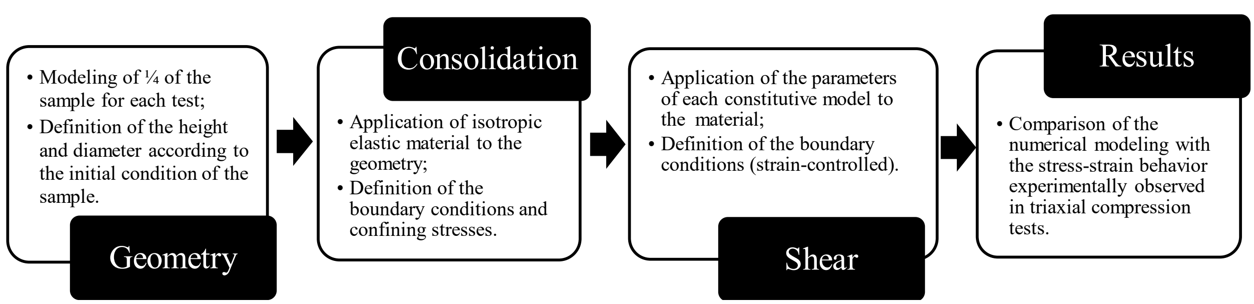

Figure 4: Methodology used in the numerical modeling

Since there was no interest in evaluating displacements that exceed the fixed limits of the numerical model, first-order horizontal and vertical displacement restrictions were adopted as boundary conditions. A quadratic mesh with elements of 0.01 m was applied. The mesh size was evaluated, and the same response was obtained for different refinement levels and for a single element calibration. Figure 5 shows the boundary conditions used for the consolidation and shear phases.

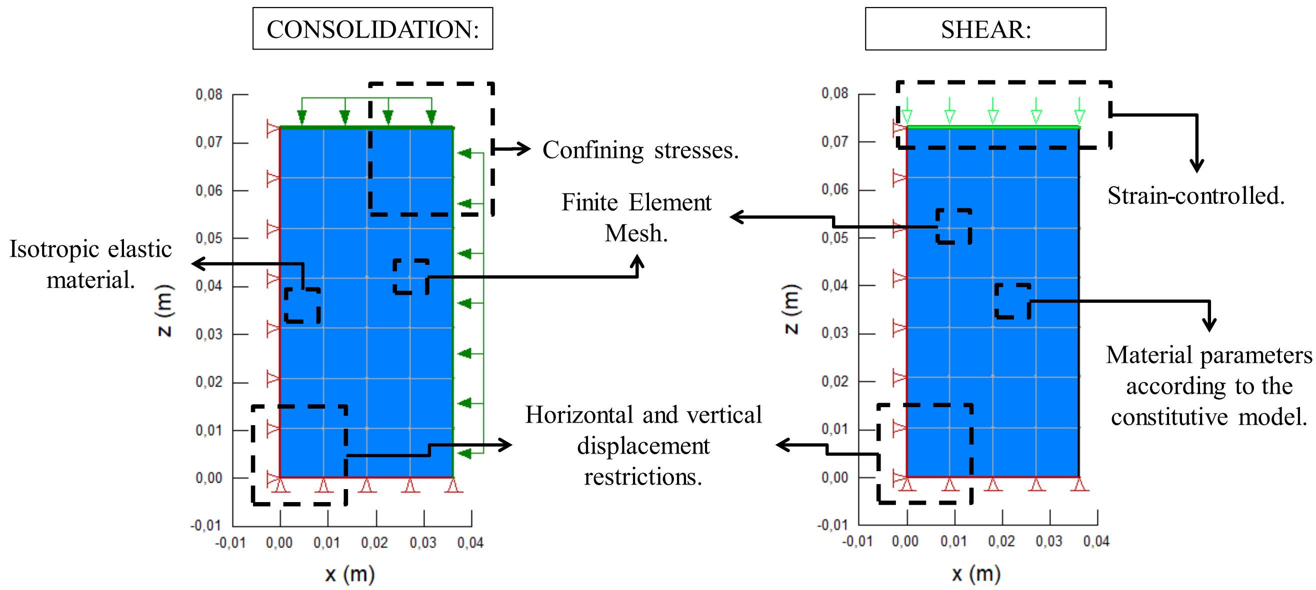

Figure 5: Boundary conditions applied for the consolidation and shear phases

Results

Mohr-Coulomb and modified Cam-Clay models

Figure 6 presents the calibration results of drained triaxial compression tests with the Mohr-Coulomb and Modified Cam-Clay models. Figure 6(a) to Figure 6(c) shows the results of a dense specimen, and Figure 6(d) to Figure 6(f) shows the results of a loose specimen.

The Mohr-Coulomb model could not correctly reproduce volumetric strains of either loose or dense samples, indicating its limitations in terms of perfectly plastic response. Also, the model could not reproduce the dilative behaviour, resulting in the absence of peak strength under drained loading for the dense sample and no simulation of the ductile behaviour for the loose sample.

The Modified Cam-Clay could accurately represent the volumetric strains, as well as the brittle and ductile behaviour for both sample conditions. Notably, the dilative behaviour for the dense sample was simulated with an OCR = 6.5 since the model uses the OCR to simulate the dilatancy effect. In terms of stress path, both models simulated the drained condition.

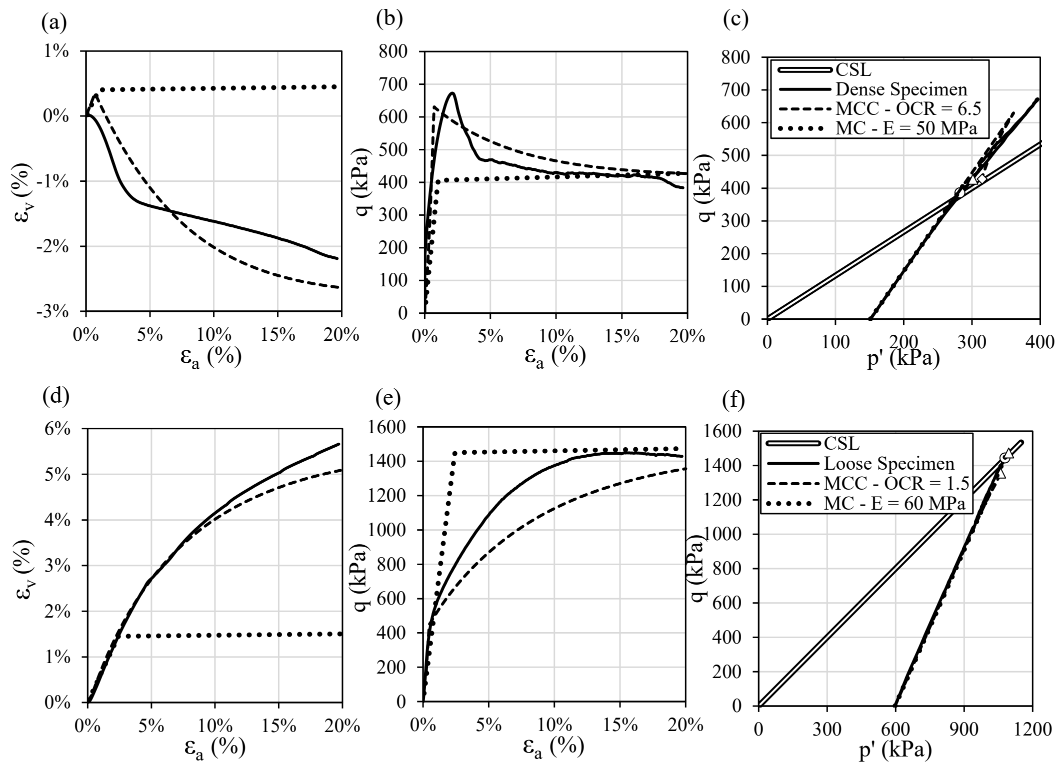

Figure 6: Calibration results – CID triaxial tests

Figure 7 shows the calibration results of undrained triaxial compression tests for the two constitutive models. Figure 7(a) to Figure 7(c) shows the results of a dense specimen, and Figure 7(d) to Figure 7 (f) shows the results of a loose specimen.

The Mohr-Coulomb model could not reproduce the experimental porewater pressure response, indicating higher values for the dense specimen and lower values for the loose specimen. Regarding stress-strain behaviour, the model showed the same perfectly plastic response observed in the CID tests and, in the $q - p'$ plane, could not simulate the dilative and contractive behaviours, originating a vertical stress path in both cases.

Considering the porewater pressure calibration, the Modified Cam-Clay model showed accurate predictions for the loose sample. However, even though the model could predict the final state of the dense sample under undrained loading, it resulted in considerably high positive porewater pressure before dilation and a different stress path from the real one. As in the CID calibration, the model reproduced the ductile behaviour of both samples. Due to its limited capacity to replicate the effects of dilatancy, the model could not correctly reproduce the dilative behaviour of the dense sample and did not recreate the behaviour change on the stress path of the loose sample, which tends to dilate in the end.

Compared to the Mohr-Coulomb, the Modified Cam-Clay model could better reproduce the red mud behaviour under drained and undrained loading. Also, it was evident that the MCC model showed a better response to the CID test since it overestimated the porewater generation for the dense sample in the CIU test and could not simulate the dilative behaviour in the stress path.

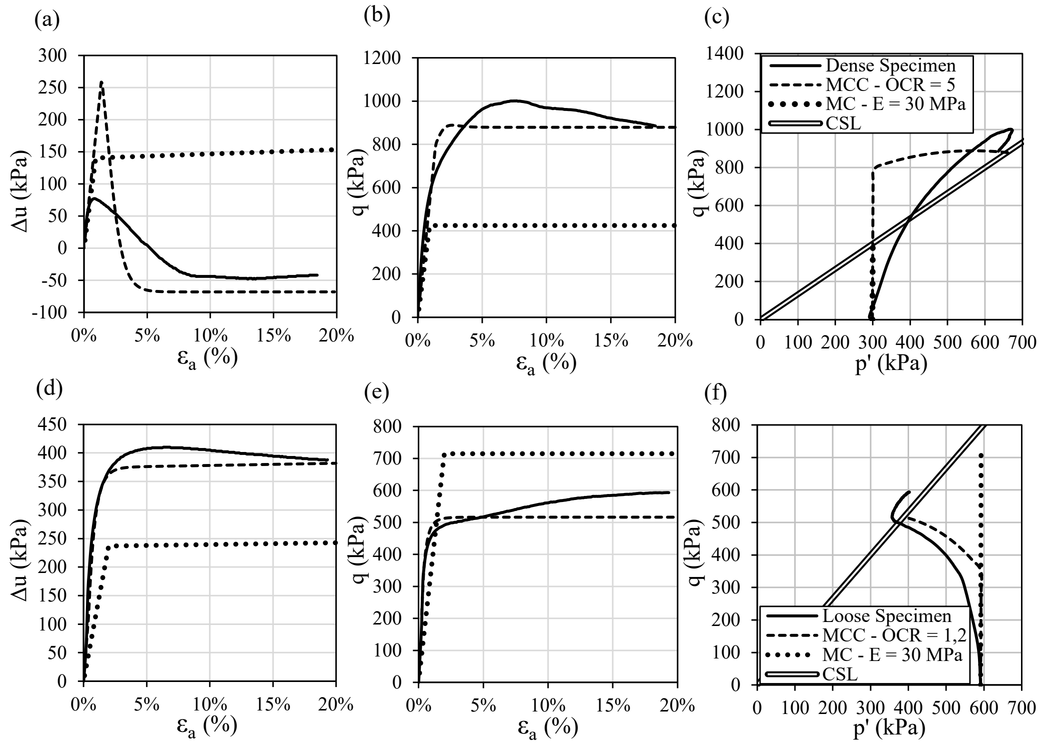

Figure 7: Calibration results – CIU triaxial tests

Modified Cam-Clay calibration and degree of compaction

The numerical analysis results were used to understand the in-situ behaviour of the compacted red mud. As mentioned in the Constitutive models parametrization, the OCR was adjusted to obtain the best fit to the experimental data when modeling the triaxial compression tests with the Modified Cam-Clay. During the process, it was observed that the OCR would change considerably with the red mud behaviour. Specimens with dilatant behaviour would calibrate with high over-consolidation ratios (OCR $\geq$ 5), while specimens with contractive behaviour would calibrate with low over-consolidation ratios (OCR < 2.5).

The OCRs obtained through the analyses were plotted against the specimens' initial void ratios (before consolidation). Figure 8 shows that, except for one specimen consolidated to 300 kPa, all the samples that showed high OCR (prone to dilatant behaviour) had an initial void ratio below 0.9. When comparing the calibration results with the main range of void ratio defined after compaction quality control tests in the dry stack (the gray region in Figure 8), it is identified that the red mud will mainly respond as normally consolidated (prone to contractive behaviour) from a confining stress of 150 kPa. Therefore, to assure over-consolidation and reduce deformations until 600 kPa of mean stress, the material should be compacted to a void ratio below 0.9, which is equivalent to a degree of compaction of 104.5% of the Standard Proctor.

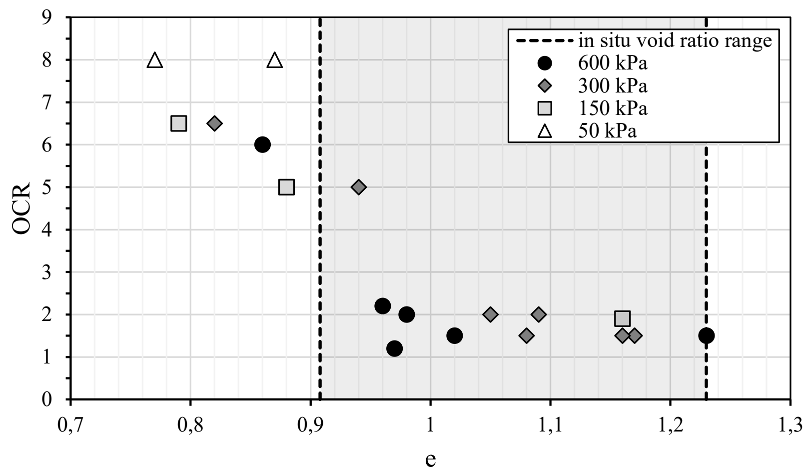

Figure 8: Void ratio variation according to the OCR used for Modified Cam-Clay calibration

Conclusion

The results from the Mohr-Coulomb model confirmed the limitations of the model's perfectly plastic response. It could not adequately recreate the ductile and brittle behaviours of the material in any numerically simulated tests. On the other hand, the Modified Cam-Clay could simulate both behaviours, considering high over-consolidation ratios (OCR $\geq$ 5) for the dilatative specimens and low over-consolidation ratios (OCR < 2.5) for contractive specimens. Such constatation is because the model uses the OCR to simulate the over-consolidation effect, and adopting a high OCR value is necessary to simulate the dilatative behaviour.

The Modified Cam-Clay showed a better response when compared to Mohr-Coulomb. Nevertheless, the results showed the Modified Cam-Clay's limitation in reproducing the undrained behaviour of the red mud dense samples. The high OCRs required to calibrate specimens consolidated under the four consolidations stresses indicated that it is possible to compact the trial dry stack in such a way that red mud will be over-consolidated (104.5% of the Standard Proctor) and prone to have a dilatant behaviour.

The responses obtained by the Modified Cam-Clay compared to the Mohr-Coulomb model are good evidence of the importance of considering the concepts of the CSSM in stress-strain analyses, especially in the mining industry, where simpler solutions are traditionally adopted. More advanced constitutive models, such as Norsand (Jefferies, 1993) and CASM (Yu, 1996), can even overcome the limitations of the Modified Cam-Clay, enabling better prediction of the undrained behaviour. However, high-quality tests are required to properly calibrate these models, and these are still difficult to obtain in commercial laboratories in Brazil.

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